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This Journal is dedicated to the following aims:

1. Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values.
2. To supply an additional medium for the publication of expository mathematical articles.
3. To promote more scientific methods of teaching mathematics.
4. To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

A Call for Aggressiveness

In our December editorial we proposed, as a choice of the more promising of two paths, that the beginnings of elementary algebra and geometry be incorporated within the college curriculum, a conservative amount of college credit being allowed for the same. The less promising path is the continuation of the practice of leaving these subjects at the mercy of the high school curriculum maker, in the face of a situation pointing with near certainty to the ultimate elimination of both algebra and geometry from required secondary courses.

Our space is too limited to record all the considerations that seem to favor a *collegiate*, rather than a *high school*, control of instruction in the initial materials of these subjects. One of them, however, may be noticed.

If the beginnings of algebra and geometry were everywhere taught by the *college* instructor, it is fairly certain that on the whole they would be taught with more thoroughness, since in practically all cases the college instructor has mathematics for his life's major interest. On the contrary, the steady decrease of required mathematics in the secondary curriculum tends to increase administrative indifference to the quality of the teaching of the mathematics that is required. It is commonly known that high school instruction in the subject is in too many instances left to those whose major concern not only is not mathematics, but is definitely antipathetic to it. In many high schools, the foot-ball coach is the sole guide of the student in his mathematics. In many others, the teacher must divide his time between mathematics and one or more of the sciences. In still others, the algebra and geometry are in the hands of an English or a history major. In not a few cases these fundamentally important subjects are left to the imperfect handling of one whose duty is not *teaching* but *administration.*

A Louisiana State University Dean told us that twenty years ago he proposed before the Southern Association of Colleges and Preparatory Schools that the first year of Latin and Greek study be incorporated in the college curriculum. His proposal met with almost unanimous disfavor; but today a majority of the colleges of the country are giving credit for the beginning portions of Latin and Greek.

Making secure in our schools the foundations of elementary mathematics is vastly more vital to civilization than is the preserving in them of the beginnings of Latin and Greek.

S. T. SANDERS.

Notes on a 17th-Century English Mathematical Manuscript*

By A. W. RICHESON
University of Maryland

Among the books and manuscripts of the James Collins Collection which was purchased for the Library of the University of Illinois in 1918 is a manuscript of mathematical exercises and formulas. This manuscript consists of 144 folios (32.5x20.5 cms.), with diagrams and problems, written in a legible late 17th-century hand. Although the manuscript is not dated, the handwriting indicates that it was written in the latter half of the 17th-century while the problems in the text set the date at about 1681. The only evidence of the original owner or author of the manuscript is found on folio 2,r, where we have in a late 17th-century handwriting "*Robert Graydon, his booke.*" There is no indication, however, where or for what purpose the manuscript was written. The present writer has carefully examined the available sources to determine whether or not anyone by the name of Robert Graydon was registered at one of the English Universities on or near the date of 1681, but without success.

There is no title page with the exception of folio 2,r which gives the list of contents. They are as follows:

> *Practicall Geometry. Decimall Arithmetick. The Extraction of Squar Roote. The use of the Table of Logarithmes. The use of the Line of Numbers. Trigonometry or the Doctrine of Straite Lined. Triangles with their application, in Measuring of Heights Distanses and Intervalls of plases both Accessible and Inaccessible. Planimetrie, or the Measuring of Planes and the Surveying of Land. Stereometrie, or the Measuring of Solids and the Gauging of vessels. Cosmographie with the use of the Globes. Geographie with the use of the Mapps. Dialling.*

The manuscript is divided into neither books nor chapters. However, the topics considered naturally lend themselves to sectional divisions, which are not numbered in any manner. The first topic, on geometry, consists of 22 folios and discusses such problems as angles, parallels, triangles, quadrangles, and figures formed by

*The present writer wishes to take this opportunity to express his appreciation to the Director of the Library of the University of Illinois for his generous cooperation in allowing access to the above manuscript.

"crooked lines." In these discussions the definitions of the terms and the elements of the figures are usually given on the upper half of the folio, while such figures as are needed to illustrate the definitions are given at the bottom of the folio. In some instances proofs of the theorems are given; in others only the theorem is stated, and applications of it shown by numerical examples. A typical theorem and proof is illustrated by the following theorem on the construction of an angle:

On a straite line given	*B.D*
and from a point on itt given as	*A*
to make a strait line angle equall to a strait line given	*F.G.H*

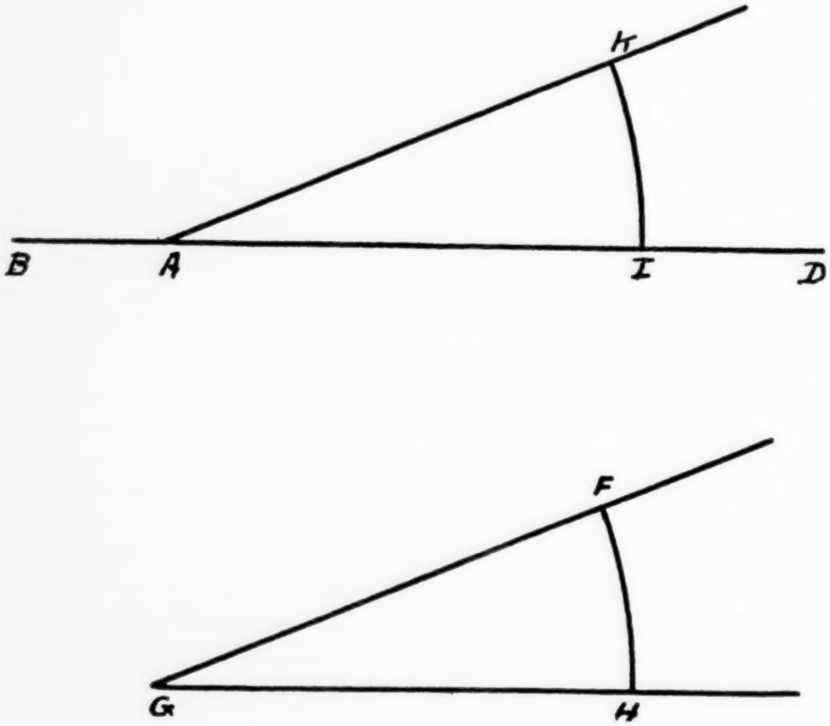

Fig. 1

Practice

On the point of the angle given	*G*
Describe an arch of a circle att any distanse as	*F.H*
Then from this given point	*A*
Describe the arch	*I.K*
Att the same distanse and take the distanse	*F.H*
and draw itt from *I* to *K*	
And draw the straite line	*A.K*
and the angle	*K.A.D*
that will equall the angle	*F.G.H*
And itt was drawn from the point *A*	
as was required	

In the discussion on triangles the writer classifies triangles as right-angled, isosceles, eqilateral, and oblique-angled. A classfication of quadrangles follows that of triangles and is as follows:

a qudrangle

- a parallelogram
  - rectangular
    - as a square
    - as an oblong or long square
  - obliqueangular
    - as a rhombus
    - as a rhomboid
- as a trapezium
  - rectangular
  - obliqueangular

The definitions of these figures are evidently taken directly from *Euclid's Elements,* since in a number of places the writer gives references similar to the following: "def. 36, El. 1st." The figures illustrating the different types of triangles are given on one full folio and those illustrating the quadrangle are given on another. These figures are apparently taken from some work of Adrain Vlacq*, since "Adrian Vlacq" is written at the bottom of these folios. On the same folio there is also a reference to a table of logarithmic sines and tangents by the same writer.

The section on "*Decimal Arithmetick*" discusses the definitions of decimals and the four fundamental operations with decimals. The

†Adrain Vlacq (Vlack)was a Dutch mathematician born at Gouda c. 1600 and died in 1655. He published six complete works on mathematics but is known principally for his work on logarithms. See *Biographie Universelle, Tome quarante-quatrieme,* (Paris, 1854) p. 1; also *Allgemeine Deutsche Biographie,* Vierzigster Band, (Leipzig, 1896), p. 86.

rules are illustrated with well-chosen numerical examples. At the close of this section there are two examples illustrating the squaring of a number, which, with a figure following the first one, are given below:

To Find the squar of 7625

```
  5
1 4   2   4
─────────────────
  7 . 6 . 2 . 5
─────────────────
4 9                 aa
  8 4               2ae   First gnomon
    3 6             ee
    3 0 4           2ae   Second gnomon
          4         ee
      7 6 2 0       2ae   Third gnomon
            2 5     ee
─────────────────
5 8 1 4 0 6 2 5     zz
```

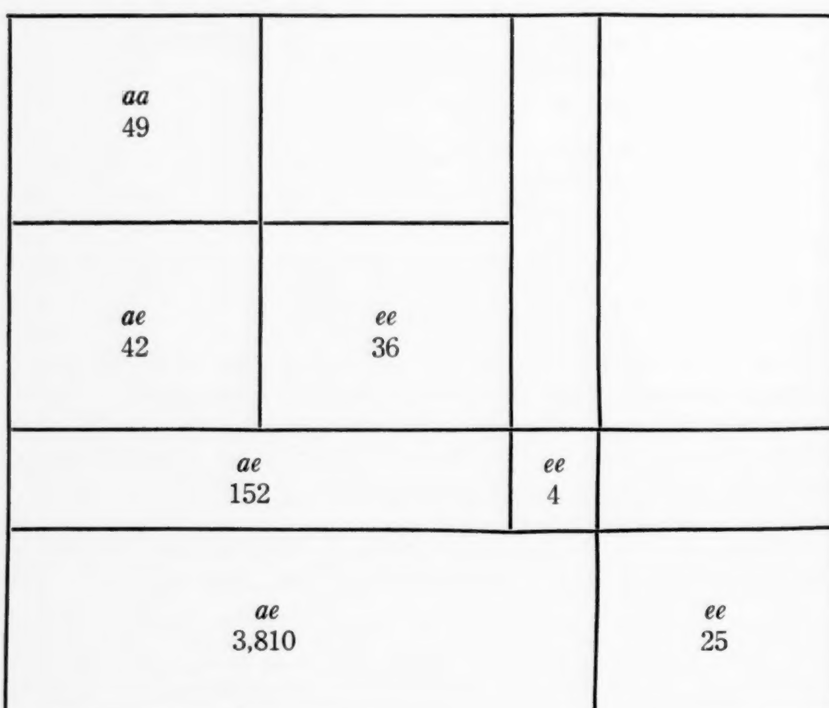

FIGURE 2

To Find the squar of 7826054

$\;5\quad\;\;5\quad\;\;1$

$\cancel{1}\,\cancel{4}\;\;6\;\;\cancel{4}\;\;2\;\;\cancel{6}\;\;0$

7 . 8 . 2 . 6 . 0 . 5 . 4

4 9
1 1 8 4
3 1 2 4
9 3 8 7 6
0 0
7 8 2 6 0 2 5
6 2 6 0 8 4 1 6

6 1 2 4 7 1 2 1 2 1 0 9 1 6(7

Subtract	4 9	*aa*
Remainder	1 2 2 4	
Divisor	1 4 8	
Subtract	1 1 8 4	$2ae \times ee$
Remainder	4 0 7 1	
Divisor	1 5 6 2	
Subtract	3 1 2 4	$2ae \times ee$
Remainder	9 4 7 2 1	
Divisor	1 5 6 4 6	
Subtract	9 3 8 7 6	$2ae \times ee$
Remainder	0 8 4 5 2 1 0 9	
Divisor	1 4 6 5 2 0 5	
Subtract	7 8 2 6 0 2 5	$2ae \times ee$
Remainder	6 2 6 0 8 4 1 6	
Divisor	1 5 6 5 2 1 0 4	
Subtract	6 2 6 0 8 4 1 6	$2ae \times ee$

The section on arithmetic is followed by "*The Use of the table of logarithmes.*" Definitions of logarithms and rules for the use of the tables and methods of finding the characteristic are given. There is

no mention of a name for the terms characteristic and mantissa, nor is the -10 written after the logarithm when the characteristic is negative.

The text then passes on to the study of angular measurements, showing the four fundamental operations with angles. At this point a number of theorems on triangles, given to aid in the trigonometric solution of the triangle, are stated. For the solution of triangles by trigonometry no definite laws are stated, but the different cases are illustrated by numerical solutions. In the case of the triangle where two sides and an angle opposite one of these sides is given, only one solution is worked out, whereas, as a matter of fact, there are two solutions. Considerable attention is given to the solution of right triangles and it is here that the theory of proportion is discussed.

In the section on "*Propositions of the Julian Calander*" the topics are of the type: "The date of the year being given to find whether it is leap year." Rules are also given for determining the day of the week on which each month begins and also for determining the date of Easter.

On the topic "The finding of the superficiall content of any plaine figure in inch measure" twenty different types of figures are considered. Seldom does the manuscript give the rule for the solution, but merely an illustrative example showing the method used. In determining the content of the trunk of a tree, the volume is found in cubic inches by a method of approximation.

In surveying, the area of a parcel of land is determined by taking the plat of the piece of ground and dividing it into a number of triangles and then summing the areas of these triangles. In case one of the sides of the plat is curved, the area of this three-sided figure is found by an approximate method.

Folios 113 to 144 are taken up with a discussion of geography and dialing. On folio 113,r a neatly drawn figure of the earth showing the zones, is given. In a note at the bottom of the folio is the following notation: "Fifth chapter of Clavarium, his Geography."† The discussion continues with numerous examples, charts, maps, and diagrams illustrating the problems in this topic.

In its treatment of dialing the manuscript gives a lengthy discussion on the construction and use of the different types of dials. A number of figures are given showing the methods of construction, and on folio 122,r an elaborate figure is given showing the construction of

†This reference is probably to Christopher Clavius, a German mathematician, born at Bamberg in 1537 and died in 1612. In 1581 he went to Rome and was employed by Pope Gregory XIII to reform the calendar. He published a number of elementary works on mathematics and astronomy. See *Biographie Universalle, Tome huitieme,* (Paris, 1854), p. 379.

a horizontal line upon the upright plane of the quadrant. At the close of the topic the manuscript gives a method for constructing the horizontal projection of the sphere.

The contents and subject matter of the manuscript would seem to indicate that the manuscript followed a definite course of instruction given at some college or university. Frequently the topics overlap each other and there is a certain amount of repetition in the type of problems, examples, and theorems. In many places there is no attempt to prove the theorems nor even to state them, but merely to give one or two numerical examples involving some unstated rule, as in the discussion of the trigonometric solution of the triangle. The definitions are, for the most part, given in simple language and are followed by numerous examples, figures, and diagrams illustrating the definitions and text matter. On the whole, however, the manuscript is very well written.

A Comparison of Methods for Finding the Interest Rate in Installment Payment Plans

By H. E. STELSON
Kent State University

There is an increasing volume of merchandise sold on time payment plans and of small loans made to consumers.* It should be of interest to the consumer to know the cost of these loans or the interest rates involved and there are at present some fairly satisfactory ways of approximating the interest rates.

Perhaps the most widely used method for computing the interest in installment plans is typically presented in Stone, Mallory and Grossnickle's "*A Higher Arithmetic.*"†

From the buyer's point of view the extra charge of the installment price over the cash price may be considered interest charged by the seller on the total of the periodic payments. For example, suppose that a gas stove is bought from a well known mail order house. It is marked at $20.50 cash or $3.00 down and $4.00 per month for five months. The difference between the total installment price and the cash price is $2.50. The buyer may consider this $2.50 as interest charged him for a series of loans amounting to $20.00. There are five loans of $4.00 each for one, two, three, four and five months respectively. These loans are equivalent to a single loan of $4.00 for $(1+2+3+4+5)$, 15 months. If $2.50 is charged for the use of $4.00 for 15 months, the equivalent interest rate is 50%.

A summary of this method of finding the interest rate is as follows:

1. Subtract the cash price from the total installment price to find the extra cost by installments.

2. Find the total of the series of weeks or months the payments run.

3. In the formula $i = prt$, substitute the amount of each periodic payment for p, and the total of the periods of payments expressed in years for t, and then solve for r. Express as a per cent.

*See Evans Clark—*Financing the Consumer*, Harper Bros. Co.

†*A Higher Arithmetic* by Stone, Mallory and Grossnickle. Benj. H. Sanborn & Co., page 270. *Champion Arithmetic* by Brown *et al.*, Row Peterson & Co., p. 357. *The Arithmetic of Business* by McMackin *et al.*, Ginn & Co., p. 328.

This formula may be expressed in general terms

(1) $$L = \frac{(m-K)2}{(m+2S-1)m} \quad \text{where}$$

$L \equiv$ Periodic interest rate.

$m \equiv$ The number of payments.

$K \equiv \frac{V-D}{R}$ where V is the cash price, D is the down payment and R is the periodical payment.

$S \equiv \%$ of R in a fractional last payment.

If the last payment is full,

(2) $$L = \frac{(m-K)2}{m(m+1)} .$$

A second method is given by Louis N. Robinson.‡ This method considers the penalty for the time plan as paid at the end of the installments. Consider the following example:

An article costs \$20.00 cash or \$4.00 down and \$4.00 per month for four months and \$2.50 the fifth month.

In this case Mr. Robinson states that the buyer pays \$2.50 the fifth month because he is not able to pay the remaining \$16.00 at the time of purchase. He therefore owes \$16.00 for the first month, \$12.00 for the second month, \$8.00 for the third and \$4.00 for the fourth month. This is the same as owing \$40.00 for one month. A charge of \$2.50 for the use of \$40.00 for one month gives a rate of 75%.

This method of computing interest rates may be expressed by the formula

(3) $$L = \frac{(m+S-K-1)2}{(h+1)(2K-h)}$$

where h is the integral number of times R is contained in $V-D$.

If the last payment is full

(4) $$L = \frac{(m-K)2}{(h+1)(2K-h)} .$$

‡See articles by Louis N. Robinson in *American Economic Review*, June, 1931, p. 233; and Lewis A. Froman in *Harvard Business Review*, January, 1933.

If $K = h$ and $S = 1$

$$L = \frac{(m-K)2}{K(K+1)}. \tag{5}$$

A very simple formula may be obtained by means of the relation: Penalty for time plan = cash value minus the down payment times the periodic rate time one-half the payment period. The statement may be expressed

$$L = \frac{(m+S-K-1)2}{mK}. \tag{6}$$

If $S = 1$

$$L = \frac{(m-K)2}{mK}. \tag{7}$$

The formulas which have been presented thus far do not find the simple interest rate, but only give an approximation to the rate. This fact has not usually been made clear when these formulas have been presented.

In considering the simple interest rate, consideration must be given to the time that the payments are equated. If the equated time is at the end of the period,*

$$L = \frac{(m+S-K-1)2}{m(2K-m+1)}. \tag{8}$$

If $S = 1$,

$$\frac{(m-K)2}{m(2K-m+1)}. \tag{9}$$

Comparing the equations

$$K = \frac{1}{1+L} + \frac{1}{1+2L} \cdots \frac{1}{1+mL} \quad \text{and} \tag{10}$$

$$K(1+L) = 1 + \frac{1}{1+L} + \cdots \frac{1}{1+(m-1)L} \tag{1}$$

*For approximation formulas for finding the simple interest equated at the beginning of the period see article by the author in the *National Mathematics Magazine*, February, 1935, p. 135.

where (10) is the equation with the focal date at the time of the transaction and (11) is the equation with the focal date one period later, it is apparent on dividing both sides of equation (11) by $1+L$ that a larger L will be required in (11) than in (10) since there is an additional squared factor in each denominator. Likewise it may be shown that L is less when the focal date is at the time of the transaction than at any other time.

Theorem. The simple interest rate on an installment payment plan is a minimum when the time of the transaction is taken as the focal date.

An actual comparison of the above formulas may be made in the following examples:

1. An article bought from a well known mail order house costs \$20.50 cash or \$3.00 down and \$4.00 per month for 5 months.

2. A large mail order house sells an article for \$20.00 cash or \$4.00 down and \$4.00 per month for four months and \$2.50 the fifth month.*

3. Mr. A. borrows \$92.00 from an industrial bank for which he repays \$2.00 at the end of each week for 50 weeks.

4. An automobile finance plan requires that one-half of one per cent be added to the unpaid balance after the down payment has been made for each month during which the payment plan is to extend. Payments to be made monthly over a period not greater than one year. Compare the interest rates for a six months plan.

Table of results of the yearly interest rates for Examples 1, 2, 3, 4.

FORMULAS

	(1) or *(2)*	*(3)* or *(4)*	*(6)* or *(7)*	*(8)* or *(9)*	*Simple Interest*	*Compound Interest*
Ex. 1	50%	63.2%	68.6%	63.2%	57.6%	72%
Ex. 2	91%	75%	75%	75%	68.1%	86.9%
Ex. 3	16.3%	19.2%	18.1%	19.3%	18.2%	18.8%
Ex. 4	10%	10.5%	12%	10.5%	10.3%	10.7%

*This is Mr. Froman's problem already quoted in deriving formula 3.

In comparing formula (10) with the formula for compound interest

$$K = \frac{1}{1+L} + \frac{1}{(1+L)^2} \cdots \frac{1}{(1+L)^m} \tag{12}$$

it is obvious that a larger L is always required to satisfy equation (12). Formulas (8) or (9) always give a greater result than the simple interest. Formulas (8) or (9) give rates equal to or greater than formulas (3) or (4) since

$$(h+1)(2K-h) > m(2K-m+1) \quad \text{or}$$

$$(m-k)^2-(K-h)^2 > (m-K)-(K-h) \quad \text{or}$$

$$m > K \quad \text{which is true since}$$

$$Rm+D > V \text{ and } K \geqslant h \text{ by definition.}$$

In conclusion, it may be stated that the approximation formulas (1)-(9) which have been either used with the idea that the simple interest itself or an approximation is obtained are not very close or consistent approximations. Their use can be justified only where very crude results will be satisfactory and where more intricate formulas are too difficult for the user. This may be the case in grade school arithmetic, but it should be made clear that they are approximations. Furthermore it would seem logical in view of the above theorem to use the beginning date of the transaction as the focal date since that gives a minimum. It has been quite customary to use simple interest for transactions extending over short periods and this paper has purposed to show the relation of some commonly used formulas to simple interest. It is not intended that this paper infers any superiority of simple over compound interest, but rather that the magnitude of the difference of these types of interest should be noted.

Complete Approximate Solutions of the Equation $x = \tan x$

By SIDNEY FRANKEL
Troy, New York

The solution of the equation

$$x = \tan x \tag{1}$$

is equivalent to the solution of the pair of equations

$$\begin{aligned} y &= x \\ y &= \tan x \end{aligned} \tag{2}$$

The solutions of equation (1) are therefore the abscissas of the intersections of the graphs of equations (2). The graphs of the inverse functions

$$y = \tan x$$

$$y = \tan^{-1} x$$

are symmetrically situated with respect to the graph of

$$y = x$$

and therefore intersect the latter at the same points; hence the solution of equations (2) is the same as the solution of

$$\begin{aligned} y &= x \\ y &= \tan^{-1} x \end{aligned} \tag{3}$$

so the solution of equation (1) is equivalent to the solution of

$$x = \tan^{-1} x \tag{4}$$

This is also indicated by taking the arc tangent of both sides of equation (1).

The discussion will be limited at first to positive solutions. The expansion of

$$y = \tan^{-1} x$$

in the neighborhood of the point, $x=a$, is

$$\tan^{-1}x=\tan^{-1}a+\frac{1}{1+a^2}(x-a)+\ldots.$$

To a first approximation, if $|x-a|$ is small,

$$\tan^{-1}x=\tan^{-1}a+\frac{1}{1+a^2}(x-a)$$

so equation (4) becomes

$$x=\tan^{-1}a+\frac{1}{1+a^2}(x-a) \tag{5}$$

If $a=(2n+1)\pi/2$, $(n=1, 2, 3, \ldots.)$, then there is a neighborhood of every a which contains a solution of equation (4). Therefore, let $a=(2n+1)\pi/2$, $(n=1,2,3,\ldots.)$. If we solve equation (5) for x, the solution for any particular value of n is ambiguous, since $\tan^{-1}a$ has an infinite number of values for any value of a. In other words, if we restrict $\tan^{-1}a$ to its principal value, equation (5) must be re-written,

$$x=k\pi+\tan^{-1}a+\frac{1}{1+a^2}(x-a) \tag{6}$$

where k is a positive integer. We proceed to determine k. We note first from the graphs of equations (2) that the first solution is $x=0$; the second solution lies between $x=\pi$ and $x=3\pi/2$; the third solution lies between 2π and $5\pi/2,\ldots.$; in general, the $(n+1)$st solution lies between $n\pi$ and $(2n+1)\pi/2$. Evidently, then, except for the case $x=0$, which we will not consider, $\tan x>1$ at an intersection point, so, for such a point

$$n\pi+\pi/4<x<n\pi+\pi/2 \qquad (n=1,2,3,\ldots.); \tag{7}$$

i. e.,

$$|x-a|=|x-(2n+1)\pi/2|<\pi/4 \qquad (n=1,2,3,\ldots.).$$

The remainder after the term, $\tan^{-1}a$, in the Taylor's expansion for $\tan^{-1}x$ is

$$(x-a)\frac{d}{dx}[\tan^{-1}x]\Bigg|_{x=x_1}$$

where x_1 lies between a and $x-a$. The remainder, therefore, is

$$\frac{x-a}{1+x_1^2};$$

thus, by equation (6),

$$x=k\pi+\tan^{-1}a+\frac{x-a}{1+x_1^2},$$

so

$$|x-k\pi-\tan^{-1}a|=\left|\frac{x-a}{1+x_1^2}\right|<\frac{\pi/4}{1+(5\pi/4)^2}<\pi/4$$

since $\qquad x_1>(3\pi/2-\pi/4)$. Thence

$$-\pi/4<(k\pi+\tan^{-1}a)-x<\pi/4 \tag{8}$$

Adding the inequalities (7) and (8),

$$n\pi<k\pi+\tan^{-1}a<n\pi+3\pi/4 \quad . \tag{9}$$

Since $\qquad a>5\pi/4,$

therefore, $\qquad \pi/4<\tan^{-1}a<\pi/2$

Combining the last inequality with (9), we get, after a slight rearrangement,

$$-\pi/2<(k-n)\pi<\pi/2$$

i. e.,

$$-\tfrac{1}{2}<(k-n)<\tfrac{1}{2}$$

But $k-n$ is an integer; we have, therefore,

$$k=n; \tag{10}$$

thus, equation (6) becomes

$$x=n\pi+\tan^{-1}a+\frac{1}{1+a^2}(x-a).$$

Solving for x, we have, finally,

$$x=[1+1/a^2][n\pi+\tan^{-1}a]-1/a \tag{11}$$

where $a=(2n+1)\pi/2$ $(n=1,2,3,\ldots.)$, and the principal value of $\tan^{-1}a$ is to be used.

Another approximation may be obtained as follows: Let

$$y = x - \tan^{-1}x \tag{12}$$

The problem is to find x when $y=0$. The expansion of the right member of equation (12) in a neighborhood of infinity is easily obtained in the following manner: From equation (12),

$$\frac{dy}{dx} = 1 - \frac{1}{1+x^2} = \frac{1}{1+1/x^2} = 1 - \frac{1}{x^2} + \frac{1}{x^4} - \frac{1}{x^6} + \ldots.$$

provided $|x|>1$. Integrating term by term,

$$y = c + x + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \ldots, \quad |x|>1. \tag{13}$$

From equation (12, using now the principal determination of $\tan^{-1}x$

$$\lim_{x\to\infty} (y-x) = (2n+1)\pi/2, \qquad (n=1,2,3,\ldots.)$$

and from equation (13),

$$\lim_{x\to\infty} (y-x) = c;$$

i. e.,

$$c = (2n+1)\pi/2$$

Putting $y=0$ and solving for x in equation (13),

$$x = (2n+1)\pi/2 \; - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \ldots. \tag{14}$$

From equation (14), we have, to a first approximation, for large values of x,

$$x + 1/x = (2n+1)\pi/2$$

whence

$$x = exp[\cosh^{-1}(2n+1)\pi/4]. \tag{15}$$

Since equations (2) represent odd functions, it is clear that the negative solutions of equation (1) are numerically equal to the positive solutions obtained above.

Tables I and II show the results of calculating x by equations (11) and (15) respectively for $n=1,2,3$. Five-place logarithms were used. After such values have been obtained, more refined results

may be obtained as indicated below. Call x and y the calculated values of x and $\tan^{-1}x$ respectively, and let x_1

n	x	$n\pi+\tan^{-1}x$	$\frac{\lvert x-(n\pi+\tan^{-1}x)\rvert}{x}\cdot 100$
1	$1\cdot431\pi$	$1\cdot430\pi$	0.07%
2	$2\cdot458\pi$	$2\cdot459\pi$	0.04%
3	$3\cdot4709\pi$	$3\cdot4709\pi$	

TABLE I

n	x	$n\pi+\tan^{-1}x$	$\frac{\lvert x-(n\pi+\tan^{-1}x)\rvert}{x}\cdot 100$
1	$1\cdot429\pi$	$1\cdot430\pi$	0.07%
2	$2\cdot458\pi$	$2\cdot459\pi$	0.04%
3	$3\cdot470\pi$	$3\cdot471\pi$	0.03%

TABLE II

and y_1 be the respective values of x and $\tan^{-1}x$ which satisfy equation (4), so that

$$y_1-x_1=0.$$

Assume that x_1-x and y_1-y are small, and write

$$x_1-x=dx$$

$$y_1-y=dy$$

Then

$$y_1-x_1=y+dy-x-dx=0$$

and since

$$dy=\frac{1}{1+x^2}dx$$

we get

$$(y-x)-\left(\frac{1}{1+1/x^2}\right)dx=0$$

whence

$$dx=[\tan^{-1}x-x][1+1/x^2],$$

where the proper determination of $\tan^{-1}x$ is used.

This represents the quantity which must be added to the calculated value of x to give a more accurate result.

On a Representation of the Newtonian Difference Quotients and Their Applications
An Abstract

By L. TCHAKALOFF
Sofia

A Newtonian difference quotient* with repeated arguments may be defined as follows: Let $a_0, a_1, \ldots, a_m$ be $m+1$ different numbers with the multiplicities $\nu_0, \nu_1, \ldots, \nu_m$ respectively. We form the linear expression

$$(1) \qquad \sum_{r=0}^{m} \sum_{\lambda=0}^{\nu_r - 1} A_{r\lambda} f^{(\lambda)}(a_r) = N\left[f \left| \begin{matrix} a_0 \, a_1 \ldots a_m \\ \nu_0 \, \nu_1 \ldots \nu_m \end{matrix} \right. \right] = N[f],$$

which depends on the values of the function $f(x)$ and its derivatives at the places $a_0, \ldots, a_m$, and determine the coefficients, which are independent of $f(x)$, in such fashion that

$$N[x^0] = N[x] = \ldots = N[x^{\mu-1}] = 0, \; N[x^\mu] = 1,$$

where for brevity we have set $\mu + 1 = \nu_0 + \nu_1 + \ldots + \nu_m$. The coefficients $A_{r\lambda}$ are uniquely determined by these conditions and it is easily shown that their determination reduces essentially to the resolution into partial fractions of the rational function

$$(z-a_0)^{-\nu_0}(z-a_1)^{-\nu_1} \ldots (z-a_m)^{-\nu_m}$$

If we assume further that the a_r are real and satisfy the inequalities $a_0 < a_1 < a_2 < \ldots < a_m$, we can then define (uniquely) a function $u(x)$ continuous in $[a_0, a_m]$ so that the identity

$$(2) \qquad N[f] = \int_{a_0}^{a_m} u(x) f^{(\mu)}(x)\,dx$$

holds. The graph of the function $u(x)$ is made up of arcs of parabolas of higher degree, which all lie entirely above the x-axis. By application of the mean value theorem to (2) we obtain

$$(3) \qquad N[f] = \int_{a_0}^{a_m} u(x) f^{(\mu)}(x)\,dx = \frac{1}{\mu!} f^{(\mu)}(\xi) \quad (a_0 < \xi < a_m)$$

*Cf. G. Kowalewski, Interpolation und genäherte Quadratur, Leipzig, 1932 and N. E. Nörlund, Leçous sur les séries d'interpolation, Paris, 1926.

and the problem arises to determine what can be said about the domain of variation of the number ξ if $f(x)$ belongs to a certain class of functions. We can in the first place say that to an arbitrary number ξ_0 between a_0 and a_m may be put in correspondence a real polynomial $f(x)$ so that equation (3), regarded as an algebraic equation in ξ, has the single (and simple) root ξ_0.

If however $f(x)$ belongs to the class $C_{k+\mu}$ of all real polynomials of at most degree $k+\mu$ (where k denotes a natural number), then we can at once apply my general theorems, which I have proved in a work published two years ago†. We can effectively determine in this fashion all minimal sets ξ_k which satisfy the following conditions:

1° To an arbitrary polynomial $f(x)$ of $C_{k+\mu}$ correspond at least one number ξ of the set ξ_k such that equation (3) holds, and

2° No proper subset of ξ_k has the property 1°.

The detailed proofs of these theorems and their applications to some classical formulas of approximate quadrature will be published in another journal.

†Sur la structure des ensembles linéares défintr par une certaine propriété minimale. *Acta Math.*, Bd. 63 (1934).

On the Solution of Equations in an Infinity of Unknowns in Linear Topological Spaces---An Abstract

By GOTTFRIED KÖTHE
Münster

Let λ be a complete linear coordinate space (cf. G. Köthe and O. Toeplitz, Journal f. d. reine u. angew. Math. 171 (1934), S. 193-226). We shall investigate when the following criterion of solvability holds:

(L) Let $A=(a_{ik})$ be an infinite matrix, which carries λ into itself, $\gamma=(c_1, c_2, \ldots)$ *a* given, $\varphi=(x_1, x_2, \ldots)$ an arbitrary element from λ. The infinite system of equations

$$A\varphi=\gamma \text{ or } \sum_{k=1}^{\infty} a_{ik}x_k=c_i \quad (i=1, 2, \ldots)$$

is then, and only then, solvable if for every sequence $(\mathring{u}^{(n)}=u_1^{(n)}, \mathring{u}_2^{(n)}, \ldots)$ from the dual space λ^* it is true that the weak convergence of $u^{(n)}A =(\sum_{i=1}^{\infty} u_i^{(n)}a_{i1}, \sum_{i=1}^{\infty} u_i^{(n)}a_{i2}, \ldots)$ to $(0, 0, \ldots)$ implies that $\lim_{n\to\infty} \mathring{u}^{(n)} =\lim_{n\to\infty} \sum_{i=1}^{\infty} u_i^{(n)}c_i=0$.

(For strong instead of weak convergence has (L) been set up by E. Schmidt and has been generalized to linear functionals of arbitrary metric spaces by F. Riesz and St. Banach.)

The following now holds: If λ is metric relative to strong convergence, then (L) holds if, and only if, every bounded infinite set of elements of λ contains a weak convergent subsequence. (L) holds thus for example in a Hilbert space, but does not in the space of all φ with convergent $\sum_{i=1}^{\infty} |x_i|$.

(L) holds also in certain non-metric complete coordinate spaces such as φ, ω, $\varphi+\omega$, but not in $\omega\varphi$.

For the systems of equations in $\omega\varphi$, whose matrices A satisfy the condition (L) a complete theory of solutions can be developed similar to the theory in the half finite space $\varphi+\omega$ (cf. G. Köthe and O. Toeplitz, Journal f. d. reine u. angew. Math. 165 (1931), S. 116-127).

Rejected Papers of Three Famous Mathematicians

By ARNOLD EMCH
University of Illinois

1. *Introduction.* Every scientifically minded person agrees with those who hold the opinion that in the appraisement or estimate of the value of an intellectual performance, merit, as acknowledged by one or more class or classes of critics, should be the deciding factor. There is no other science in which this assertion should appear more evident than in mathematics. And yet history records some strange exceptions to this rule, due to the fact that mathematicians also are frequently very human, just like the rest of their scientific brethren.

In the following lines I propose to discuss three outstanding examples of extremely meritorious papers which were rejected by supposedly competent critics in editorial or academic committees. This does not mean that these are the only known cases, because there is no doubt that many other instances could be mentioned in support of these curious historic events.

2. *Schläfli*: *Theorie der vielfachen Continuität.* This paper on the theory of manifold continuity was written by L. Schläfli, professor of mathematics in the University of Berne, Switzerland, between 1850 and 1852 and was submitted to the Academy of Science in Vienna in July, 1852, but was refused on account of its length. In 1854 Schläfli sent the manuscript to Crelle's Journal, which tentatively accepted it for publication, but returned it in 1856. Not until 1901 was this remarkable memoir finally published in the *Denkschriften* (proceedings) of the *Schweizerische Naturforschende Gesellschaft.*

Before giving a short account of Schläfli's accomplishment it is necessary to state that definitions of spaces or manifolds of more than three dimensions were given before. In 1844 Grassmann published *Die lineare Ausdehnungslehre,* a pioneer and classic in the theory of manifolds. A year before Cayley wrote a short paper: "Chapters in the analytic geometry of n dimensions", which appeared in his collected works Vol. IV (1843), pp. 119-127.

The *Comptes Rendus* of the French Academy of Science, Vol. 24 (1847), pp. 885-87, contains an article by Cauchy: *mémoire sur les lieux analytiques,* where we find a clear cut statement of an n-dimensional Euclidean geometry as we understand it at the present day. But it indicates rather than establishes an analytic geometry of higher dimensions.

As far as we know, Schläfli was the first to do this explicitly in a masterly fashion. Were it not for the unfortunate fact that his memoir was not published until 1901, historic accounts of the theory of hyperspaces would not ignore but would mention Schläfli's efforts in the first rank.

In this theory of multiple continuity of n dimensions, n variables $x, y, z, \ldots$; 1, 2, 3,... equations determine in succession $(n-1)$, $(n-2), \ldots$ $-$fold continua. Distance between two points $x', y', z', \ldots$; $x, y, z, \ldots$ is defined as

$$\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2+\ldots},$$

or in oblique coordinate system as

$$\sqrt{(x'-x)^2+(y'-y)^2+\ldots 2k(x'-x)(y'-y)+\ldots}$$

He studies orthogonal transformations (movements in n-space), defines angles between two lines or two hyperplanes

$$p=ax+by+cz+\ldots+hw,$$

$$p'=a'x+b'y+c'z+\ldots+h'w,$$

as

$$-\cos(\angle pp') = \frac{aa'+bb'+cc'+\ldots+hh'}{\sqrt{a^2+b^2+\ldots} \cdot \sqrt{a'^2+b'^2+\ldots}}$$

Schläfli studies the integral

$$S_n = \int\int\int^n \ldots dx\, dy\, dz \ldots$$

with boundary conditions $p_1>0, p_2>0, \ldots, p_n>0$, $x^2+y^2+\ldots+w^2<1$ There are at least $n+1$ hyperplanes necessary to bound an n-dimensional solid (polyscheme) whose measure is

$\int\int\int \ldots \int dx\, dy\, dz \ldots$ extended over this hypersolid. Higher continua (varieties) are defined by $x=\phi(f_i)$, $y=\psi(f_i)$, $z=\chi(f_i)\ldots$; $i=1, 2, 3, \ldots, m$; in which $\phi, \psi, \chi, \ldots$ are not

linear functions of $m \leqq n$ variables. He studies regular polyhedra in four and more dimensions and their metrical properties. In the second part Schläfli discusses the theory of spherical continua bounded by $x^2+y^2+\ldots<1$ and n linear homogeneous independent polynomials. Part three contains a study of hyperquadrics and their metric properties, including confocal hyperquadrics. Some of these applications are simply generalizations which are included for the sake of completeness. Schläfli himself was very well aware of the intrinsic non-importance of mere generalizations: *Wenn ich nun auch das Verdienst des Generalisierens nur gering anschlage.*

3. *Riemann*: *Commentatio mathematica qua respondere tentatur quaestioni ab Ill$^{\underline{ma}}$ Academia Parisiensi propositae.* In the year 1858 the Academy of Science in Paris proposed a prize-question, which in the original (to be very accurate) is as follows: *Trouver quel doit être l'état calorifique d'un corps solide homogène indéfinie pour qu'un système de courbes isothermes après un temps quelconque, de telle sorte que la température d'un point puisse s'exprimer en fonction du temps et de deux autres variables indépendantes.*

Riemann headed his memoir with the motto: *Et his principiis via sternitare ad majora.* Nothing was done! It was handed in July 1, 1861. The prize was refused, because the methods by which the results were obtained were not fully explained. The explicit treatment of the problem was prevented by Riemann's ill health. The memoir, which was in Latin, was published by H. Weber on pp. 391-423 in the collected works of Bernhard Riemann in 1892. In this memoir Riemann presents the foundations of manifolds of multiple dimensions and makes an ingenious application of this theory. The original manuscript was returned to the Göttingen Academy of Science on request, by the secretary of the French Academy.

The ideas involved find expression in Riemann's classic "*Probevorlesung*" at Göttingen, June 10, 1854, for habilitation as a lecturer at the university: *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen.* This was published separately by H. Weyl in 1919, including valuable comments. In this Riemann explains n —dimensional manifolds and their various geometric interpretations and applications.

Again to preserve absolute accuracy we quote the concluding statement of Riemann in the author's original version:

"Es muss also entweder das dem Raume zugrunde liegende Wirkliche eine diskrete Manigfaltigkeit bilden, oder der Grund der Massverhältnisse ausserhalb, in daraufwirkenden bindenden Kräften gesucht werden.... Es führt dies hinüber in das Gebiet einer andern Wissenschaft, in das Gebiet der Physik."

In his comments Weyl remarks (translation): "Thus Riemann's idea which tore down the barrier between geometry and physics has found its most brilliant confirmation by Einstein's theory."

3. *De Jonquières: De la transformation géométrique des figures planes.*

The *Nouvelles Annales de Mathematiques,* Vol. 3 (1864), pp. 97-111 contains a paper by De Jonquières under this title in which he states that he had sent a memoir on this subject to the Institute of France. The *Comptes Rendus* of 1859, Vol. 49, p. 542 acknowledges the receipt of such a memoir. But there never was a report made, so that it rested for nearly a quarter of a century among the Archives of of the Academy.

In the first part of the memoir is established a plane transformation in which to lines in one plane correspond in another plane curves of order n which all have a point of multiplicity $n-1$ in common. De Jonquières thus considered birational transformations between two planes before Cremona published his important results in 1863.

In a conversation with G. B. Gucia, De Jonquières mentioned the results of these investigations, whereupon Gucia expressed the desire to read an authentic copy of the memoir with a view to publishing it in a scientific journal of Italy. This was eventually done in the *Giornale di Matematiche,* Vol. 23, 1885, pp. 48-75. The title of this paper is

Mémoire sur les figures isographiques et sur un uniforme de géneration des courbes à double coubure d'un ordre quelconque au moyen de deux faisceaux correspondants de droites.

In it De Jonquières studies the net of curves of order m through $2(m-1)$ simple points and an $(n-1)$ –fold point. Then he considers three arbitrary lines $L=0, L'=0, L''=0$ so that every other line can be represented by

$$L+\lambda L'+\mu L''=0$$

With these are associated three curves $c=0,\ c'=0,\ c''=0,$ so that to

$$c+\alpha\lambda c'+\beta\mu c''=0$$

corresponds $L+\lambda L'+\mu L''=0$, with $\alpha,\ \beta$ as constants. To the intersection of two lines (L) and (L_1) corresponds uniquely the one variable intersection of the corresponding curves (C) and (C_1). In this birational transformation, which is now universally known as a De Jonquières transformation, the fundamental points are the $2m-2$ simple points and the $(m-1)$ –fold point. De Jonquières who was "capitaine" in the French navy, wrote the memoir on board the frègate le d'Assas, November 25, 1859.

In Memoriam: E. J. B. Goursat

By G. WALDO DUNNINGTON

Professor Edouard Jean Baptiste Goursat, of the Sorbonne, died November 26, 1936, after a long career of notable activity in the field of mathematics. He was born May 26, 1858, at Lanzac (Lot) and obtained his early education at the Collège de Brive, the lycée Henri IV, and the Ecole normale supérieure; he received the degree of doctor of science in 1881. His wife was Mlle Andrée Rebière.

Goursat was a fellow of secondary instruction, and lectured at the University of Paris from 1879 to 1881; at the University of Toulouse, 1881-1885; at the Ecole normale supérieure, 1885-1897. He became professor of Analysis in the faculty of sciences at the University of Paris in 1894, a position he held until his death. In the Ecole polytechnique he became *répétiteur* in 1896 and professor of differential and integral calculus in 1897.

His honors were numerous: member of the Academy of Sciences in the French Institute, chevalier of the Legion of Honor, officer of public instruction, grand prix for mathematical sciences (1886), Poncelet prize (1889), and prix Petit d'Ormoy (1891). He was a former president of the Mathematical Society of France, and laureate of the Academy of Sciences.

A fairly complete list of his papers is to be found in Poggendorff. They appeared in the Comptes rendus, Bulletin de la Soc. math. de France, Trans. Am. Math. Soc., Bull. d. Sc. math. (Darboux), Liouville's Journal, Nouv. Ann. de Math., Paris Ecole norm. Ann., Acta mathematica, and Toulouse: Fac. d. Sc. Ann.

Professor Goursat's major works were: *Leçons sur l'intégration des équations aux dérivées partielles du premier ordre* (*1890*); *du second ordre á deux variables indépendantes* (*1896-98*); *Théorie des fonctions algébriques et de leurs intégrales* (*1894*); *Cours d'analyse mathématique* (*1902-1905*) the English translation of which was done by E. R. Hedrick and Otto Dunkel, and published by Ginn & Co., Boston, 1917; *Leçons sur le problem de Pfaff* (1922). Goursat also published a revision of Briot's *Leçons d'algèbre*.

His works, embracing all the branches of infinitesimal analysis and containing numerous original conceptions, have placed him among the leading analysts of his generation.

Sectioning Students on the Basis of Ability

By FREDRICK WOOD
The University of Nevada

Many arguments have been made concerning the value of sectioning students on the basis of ability, and many reports have been made by institutions where such a system has been used. I wish to report the results of such an experiment and to make some comments upon the results.

After teaching for many years and always faced with the problem of how to present material to a class in such a way as neither to bore the brilliant ones nor discourage those poorly prepared, I found the opportunity here of giving a thorough trial to a system of grouping students according to ability.

At the University of Nevada a five-hour course in Mathematical Analysis is required of all Engineering students in the Freshman year. This course consists of the elements of algebra, trigonometry, analytic geometry, with many applications. For several years it has been customary here to divide the class into three groups. The following are the minimum requirements for classification in the respective groups.

Group 1—Algebra, one and one-half years; trigonometry; plane geometry.
Group 2—Algebra, one and one-half years; plane geometry.
Group 3—Algebra, one year; plane geometry.

At registration time, the students are assigned to the different sections by their adviser according to the grouping just indicated. During the opening class period each instructor explains to his class that the arbitrary assignment of students to groups is not final at all but quite temporary. Our schedule permits us to have three sections at the same hour which makes section readjustments easy for us and in no way affects the schedule of classes in other departments. The course begins with a very intensive review of the fundamentals of high school algebra. After two weeks an examination is given and certain adjust-

ments are made, students going to sections where they seem more likely to belong. We have several changes at this time but comparatively few later. However, adjustments are made from time to time when necessary. It should be made clear that the students enter quickly into the spirit of the game and, in the large majority of cases, ask to be transferred before the instructor suggests it. We are quite careful to avoid discouraging a student when changing him to a lower section and allow him very often to make his own decision.

Before discussing the merits of the arrangement, the grading system should be explained. For purposes of discussion let us say that there are grades of A, B, C, D, E, F, A being excellent, D the lowest passing grade, E condition, and F failure. In our sectioning system, we announce that if a student is in Group I at the end of the semester his grade will be A, B, or C; if he is in Group II at the end of the semester his grade will be B, C, or D; and if he is in Group III at the end of the semester his grade will be C, D, E, or F. Some overlapping of the grades given in the different groups is thus allowed and this takes care of many border-line cases of students who could be in either of two groups. Too much readjustment, which might interfere with the efficiency of the work, is thus prevented. This allows for enough leeway to take care of most cases where a student has a strong preference for a certain section.

The question naturally arises as to how the material is presented to the different groups in order to keep their assignments the same and to allow for transfers without difficulty. Our aim is to keep Group I somewhat ahead of Group II and Group II somewhat ahead of Group III in order to emphasize the fact that we are sectioning on the basis of ability. The effect would not be wholesome if, for example, Group I was behind Group II in the text. If a student is earnest, conscientious, and willing, he can make up the work if he enters a section which is ahead by a few topics. In fact, he is encouraged by his instructor to work ahead of his class before he is transferred. If he transfers to a section which is behind by a few topics he is able to get another chance at some topics he may understand but vaguely. In cases of absence through illness a student may be able to get needed instruction in this manner and work back to his section later when otherwise he might fail to get along at all. In Group I we take up supplementary material not in the text-book in order to allow Group II to keep up; in Group II we follow the text-book closely, taking very little supplementary material; in Group III we find it necessary to omit certain parts of the text.

Now let us discuss the merits of the system. In Group I we find a very fine spirit, the students are eager to learn, willing to do more than the instructor requires, there is plenty of friendly rivalry, they are proud of their rating, they have a certain prestige on the campus, the class is not delayed by students of little high school preparation, and, in my opinion, Group I has all advantages and no disadvantages through the sectioning arrangement. When a student realizes that the majority of the members of the class understand easily what he gets with great difficulty he will often ask to be transferred to Group II.

In Group II, as well as in Group I, we find a good attitude toward the work, the students are of somewhat the same ability, there are no exceedingly brilliant students to outshine everybody else, and there are no exceedingly weak students. The work goes on at a fair rate of speed, the essentials of the course are covered, there are good grades for the best of the members of the class, and passing grades for all. The student knows that his work is of passing grade as long as he is permitted to remain in the group. He has therefore considerable incentive to make an effort to make good. Often a student who transfers from Group I to Group II, gets his bearings after travelling at a somewhat slower pace and returns again to Group I. Also he finds quite often that his level is in Group II and will quite frankly say that he prefers a grade of B or C in Group II where the speed of the class suits him and does not desire return to Group I where the class leaves a topic before he is able to master it.

After several years of experience in teaching these different groups, we wish to state that we believe that the students in Group III are better off than if we had no sectioning arrangement. We do not at any time refer to them in any way that would indicate that we think that they lack ability. We merely make it clear that because of lack of high school preparation we find it necessary to register them in Group III and that there is no disgrace in being in the group. We do make it clear that good work in Engineering requires more than just one year of algebra and one year of geometry in high school, and that they must expect to be handicapped in college at first, but that this handicap can be removed by thorough and conscientious effort. The instructor of this group finds great earnestness in his students, close attention, and finds that his class makes slow but sure progress. Some of the students reach Group II by making strong efforts, and, of course, some withdraw from the University or transfer to another college of this University when they find that they are totally unfitted for Engineering.

It has been our experience then in sectioning students according to ability that bright students are allowed to progress rapidly and are not held back, that average students are not discouraged but find their own level and make progress there, and that the students of less ability and preparation also make progress if they are at all fitted for Engineering. We do not find discouragement in Group III because of the sectioning; if it is there it is because of other conditions. We do find a fine spirit in each section. From my own experience in teaching engineering mathematics here and elsewhere where sectioning was not used I can say that I am very much in favor of this system and our administrative officers are quite satisfied with the results here. Contrary to my belief before I tried it, I do not find myself bored or discouraged when I teach Group III; many of the students are just as earnest, faithful, and conscientious as are the students in Group I.

Because of certain conditions in the College of Arts and Science that prevent scheduling of more than one section of mathematics at the same hour we have not been able to experiment in that College. We do carry our sectioning plan into Calculus and Mechanics for Engineers and find the same satisfactory results as we do in the Freshman year.

From all the evidence that I can collect here, by the opinions of faculty and students, by the progress of students in subsequent courses, the conclusion is that for the University of Nevada sectioning according to ability in Mathematics is much to be preferred to any other method.

A Fraction Rule in Logarithms

By M. G. SCHERBERG
University of Minnesota

This fraction rule in logarithms is very likely to give the student a more permanent hold on the method for changing bases in logarithms. The rule is at once evident if we compare the identities

$$\text{Log}_a b \cdot \text{Log}_b c = \text{Log}_a c$$

$$b/a \cdot c/b = c/a$$

or

$$\text{Log}_a b = \frac{\text{Log}_a c}{\text{Log}_b c} = \frac{\text{Log}_c b}{\text{Log}_c a}$$

$$b/a = \frac{c/a}{c/b} = \frac{b/c}{a/c}$$

Thus

$$\text{Log}_5 \cancel{7} \cdot \text{Log}_{\cancel{2}} \cancel{3} \cdot \text{Log}_{\cancel{7}} 4 \cdot \text{Log}_{\cancel{3}} 7 \cdot \text{Log}_8 \cancel{2}$$

may be written

$$\text{Log}_5 4 \cdot \text{Log}_8 7$$

or

$$\text{Log}_5 7 \cdot \text{Log}_8 4$$

To differentiate $\text{Log}_x 2$, write it $\dfrac{\text{Log}_e 2}{\text{Log}_e x}$; to evaluate it $\dfrac{\text{Log}_{10} 2}{\text{Log}_{10} x}$.

The one great danger in this symbolism is that the student will attempt to carry it too far. Thus $\text{Log}_4 6 \neq \text{Log}_2 3$ for $\text{Log}_4 5 \cdot \text{Log}_5 6 \neq \text{Log}_2 5 \cdot \text{Log}_5 3$. The whole number must be cancelled, not part of it.

Mathematical Notes

Edited by
L. J. ADAMS

Robert C. Yates, University of Maryland, has recently been promoted to an associate professorship at the University of Maryland.

Professor J. N. Michie, head of the Department of Mathematics in Texas Technological College and Fellow of the Texas Academy of Science, addressed the Academy on November 13. The title of his address was: *A simple modification of the root squaring process* (*Graeffe's method*) *for the numerical solution of algebraic equations when there are four or more pairs of complex roots in a given equation.* This paper was an extension of what appeared in the April number of the *American Mathematical Monthly* by Mr. Hausman of New Haven, Connecticut.

The thirty-second annual High School Conference of Illinois was held at the University of Illinois, Urbana, on November 5, 6 and 7, 1936. The complete program of the mathematics section follows:

1. "*The Perennial Task of Re-thinking Mathematics*"—H. W. Bailey, Assistant Professor of Mathematics, University of Illinois.
2. "*Making Needed Adjustments for Meeting Mathematical Needs of High School Pupils*"—A. W. Clevenger, High School Visitor, University of Illinois.
3. "*Intensifying the Accuracy of and Broadening the Scope of the Student's Mathematical Horizon*"—W. A. Snyder, Head of the Mathematics Department, New Trier Township High School, Winnetka.
4. Discussion—Leader: C. M. Austin, Head, Mathematics Department, Oak Park and River Forest Township High School, Oak Park.
5. "*Salvaging Geometry in the High School*"—H. H. Foster, Professor of Education, Beloit College, Beloit, Wisconsin.
6. "*The Present Crisis in the Teaching of Elementary Mathematics*"—William Betz, Specialist in Mathematics for the Schools of Rochester, New York.

   Organization for participation in the State Curriculum Study.
7. "*Innovations in Illinois High Schools, as Reported by Principals*"—G. M. Worbois, Service Fellow in Education, University of

Illinois; and K. C. Dillow, Service Fellow in Education, University of Illinois. (Material presented in mimeographed form.)

8. "*Working Relationships Within the High School and Between the School and the Community*"—C. C. Stadtman, First Assistant Superintendent of Public Instruction; and C. W. Sanford, Principal, University High School, Urbana.
9. Educational and Vocational Guidance of High School Pupils:
   (1) "*Educational Guidance as Viewed by the Principal of a Large High School*"—Olice Winter, Principal, Lake View High School, Chicago.
   (2) "*Some Specific Suggestions Relative to Planning a Program for the Educational and Vocational Guidance of High School Pupils*"—Arthur W. Clevenger, High School Visitor, University of Illinois.
10. "*Making Special Provisions in the High School for Meeting the Needs and Interests of Non-College Preparatory Pupils*"—H. D. Trimble, Assistant High School Visitor, University of Illinois; Paul E. Belting, Assistant Superintendent of Public Instruction; and Miss Lita Bane, Head, Department of Home Economics, University of Illinois.
11. "*Meeting the Needs of the Superior Student*"—Walter S. Monroe, Professor of Education, Director, Bureau of Educational Research University of Illinois.
12. "*Mass Education in the Secondary Schools and Resulting Problems*"—William Betz, Specialist in Mathematics for the Schools of Rochester, New York.

The chairman and secretary of the mathematics section were E. G. Hexter and C. N. Mills, respectively.

Dr. Ernest Barnes Lytle, for many years associate professor of mathematics in the University of Illinois, died September 5, 1936, after a long illness.

Dr. George A. Miller, professor of mathematics, emeritus, in the University of Illinois, received in June, 1936, the LL.D. degree from Muhlenberg College, Allentown, Pennsylvania. Prof. Miller received the A. B. degree from Muhlenberg in 1887; he is probably the only American who has been co-editor of the "*Encyclopedie des Sciences Mathematiques.*"

Professor H. C. Van Buskirk, California Institute of Technology, reports the following news concerning members of the mathematics department of the Institute:

Dr. A. D. Michal, Dr. Morgan Ward and Dr. Angus Taylor attended the meeting of the American Mathematical Society at Cambridge, Massachusetts, August 28-31, 1936

Dr. Harry Bateman spent the summer in England and attended the following meetings:

Review at Trinity College, Cambridge,
June 23—years 1898-1905.

Centenary of the University of London,
June 27-July 3.

International Congress of Mathematicians, Oslo, Norway,
July 13-18.

International Union of Geodesy and Geophysics, Edinburgh,
September 14-26.

Professor C. D. Smith, Mississippi State College, reports the following news items from that institution:

Mr. F. P. Welch is on leave for the purpose of completing requirements for his Doctor's degree at the University of Illinois.

Mr. R. A. Smith was appointed temporary instructor in the absence of Mr. Welch.

Mr. L. S. Lundy was transferred to the Discipline Department.

Mr. W. O. Spencer was appointed temporary instructor.

Assistant Professor Hal Fox has been promoted to an Associate Professorship.

The following two courses open to graduate students have been added to the list of course offerings:

Theory of Numbers by Mr. W. E. Cox.

Advanced Algebra by Dr. Arthur Olliver.

From Lehigh University Professor Tomlinson Fort reports the following:

Assistant Professors G. E. Raynor and C. A. Shook were promoted to associate professorships.

Dr. S. S. Cairns is on leave of absence for the year to study at the Institute for Advanced Study at Princeton.

Dr. E. H. Cutler has returned to Lehigh from a year spent at the Institute for Advanced Study at Princeton.

Professor G. A. Bliss, The University of Chicago, reports the following news items from that university:

Professor L. E. Dickson received an honorary Sc.D. degree at the Harvard Tercentenary celebration in September.

Professor W. D. MacMillan of the Department of Mathematical Astronomy retired last June.

Dr. W. T. Reid is spending the autumn Quarter at the Institute for Advanced Study during his quarter out of residence.

Dr. D. M. Dribin (Ph.D. Chicago, 1936) holds a National Research Fellowship and is spending the year at the Institute for Advanced Study in Princeton.

The following appointments as instructors have been reported for former students of the Department: Dr. M. R. Hestenes, University of California at Los Angeles; Dr. Ralph Hull, University of Michigan; Dr. Max Coral, Wayne University, Detroit; Dr. M. Gweneth Humphreys, Sophie Newcomb College, New Orleans; Dr. Ruth Mason, University of Illinois; Dr. O. K. Sagen, Iowa State University; Mr. Carl Denbow, Ohio University; Mr. F. A. Valentine, University of Tennessee; Miss Kathryn Cardwell, Baylor University.

Dr. Bengt Stromgren has been appointed to an assistant professorship in Astrophysics.

Professor G. H. Hardy gave a lecture at the University of Chicago on November 13 on the subject, "*Ramanujan and the theory of primes.*"

Mr. Zens L. Smith has been appointed assistant professor in mathematics with duties in the University High School and the General Physical Science Survey course in the college.

Dr. Nathan Jacobson (Ph.D. Princeton, 1934) is spending this year as National Research Fellow at the University of Chicago.

Professor T. Vijayaraghavan gave two lectures at the University of Chicago on November 25 and 27 on "*Elementary Tauberian theorems*" and "*On the increase of real solutions of algebraic differential equations.*"

Professor I. M. H. Etherington, Hon. Secretary of the Edinburgh Mathematical Society of Scotland, submits the current annual program of that organization, as follows:

1. Friday, November 6.
   Presidential Lecture: "*Modern differential geometry,*" by Dr. H. S. Ruse, M. A., D.Sc.
2. Friday, December 4.
   Research papers.
3. Friday, January 15.
   "*School Mathematics,*" by Mr. William Taylor, M. A.

4. Friday, February 5, at Dundee.
   "Three quadric surfaces," by Mr. W. L. Edge, M. A.
5. Friday, March 5.
   Research papers or mathematical notes.
6. Saturday, May 8, at Aberdeen.
   "The solution of Waring's Problem," by Professor E. Maitland Wright, M. A., D.Phil.
7. Saturday, June 5, at St. Andrews.
   Lecture by Dr. A. C. Aitken, F. R. S.

The meetings in November, December, January and March will be held at 6:15 p. m. in the Mathematical Institute, 16 Chambers Street, Edinburgh. Details concerning the other meetings will be announced in due course.

At the opening meeting on November 6th, Professor R. O. Street, M. A., D.Sc., was elected President, and Mr. George Lawson, M. A., Vice-President.

The three hundred thirty-seventh meeting of the American Mathematical Society was held at the University of California at Los Angeles, November 28, 1936. Luncheon was served for members and their guests in the faculty Dining Room of Kerckhoff Hall. Thirteen research papers were presented, and thirteen others by title only.

The three hundred thirty-sixth meeting of the American Mathematical Society was held at the University of Kansas, Lawrence, Kansas, on Friday and Saturday, November 27-28, 1936. By invitation of the Committee on Program three addresses were given. On Friday afternoon Professor H. L. Rietz spoke on *Some topics in sampling theory,* and Professor Constantin Caratheodory on *Bounded analytic functions.* Professor L. R. Ford addressed the Society Saturday morning on *Fractions.* Some nineteen research papers were presented by members of the society.

The twenty-first annual meeting of the Mathematical Association of America was scheduled to be held at Durham and Chapel Hill, North Carolina, on Thursday, December 31, 1936 and Friday, January 1, 1937, in conjunction with the meeting of the American Mathematical Society. The next issue of the National Mathematics Magazine will contain the program of these meetings.

Book Reviews

Edited by
P. K. SMITH

First Year College Mathematics. By M. A. Hill, Jr. and J. Burton Linker, Henry Holt and Company, New York, 1936. x+436 pages. +155 pages of tables.

This text is divided into three parts:

I. *Algebra and Trigonometry.*
II. *Analytic Geometry.*
III. *Mathematics of Finance.*

The authors advise Parts I and II for students of the general arts and sciences, Parts I and III for commerce majors and all three parts for engineering students. The subject matter of the text proper is followed by answers to all of the odd-numbered problems, and seventeen tables covering one hundred and fifty-five pages.

Part I is an attempt to unify college algebra and plane trigonometry. Its contents are suggested by the titles of its chapters, which are: Introduction, Functions, Graphical Representation of Functions, Factoring, Fractions, Exponents and Radicals, Logarithms, Solution of Right Triangles by Logarithms, Linear Equations, Quadratic Functions and Equations, Equations of Higher Degree, Functions of Multiple Angles, Oblique Triangles, Complex Numbers. This list reveals what one finds to be true, viz., that a few of the numerous topics usually included in a college algebra text are omitted. For example, permutations and combinations are omitted entirely, while probability is very briefly treated in connection with the mortality tables and life insurance in Part III. Partial fractions are also omitted, but that topic is often postponed until its use in integral calculus, anyway. As a matter of fact, the extent of the loss by omissions is debatable, since the entire conglomeration of topics usually included in college algebra texts is very seldom completely covered in a semester course in the subject.

The constructive work in unification of algebra and trigonometry is brought about by "introducing the general trigonometric ratios early in the course and from that point on employing them indiscriminately with the algebraic symbols in the study of graphs, factoring, fractions, etc." It is the opinion of this reviewer that attempts

to unify any two given subjects usually result in advantage to one subject at the expense of the other. Part I of this text is no exception, with trigonometry gaining at the expense of algebra. Certainly students who study Part I thoroughly will be well equipped in the matter of manipulating trigonometric functions, a necessity in the differential and integral calculus.

Parts II and III are the standard courses in analytic geometry and mathematics of finance. The authors offer, in these two parts, a very teachable presentation of the elements of the subjects. The explanations are unusually lucid, written in nearly conversational style. The graphs and diagrams of Part II are both numerous and readable. Part III is somewhat brief, comprising only ninety pages, but the material given is selected with care and will furnish the student with a good introduction to the subject.

There are approximately three thousand problems in the entire book. They are not separated into groups, but they are well arranged in order of difficulty. The format is excellent, far above the average in freshman mathematics texts.

Those who are interested in presenting first year college mathematics in such a way as to include:

1. unification of college algebra and trigonometry,
2. standard course in analytic geometry,
3. introduction to mathematics of finance,

would do well to examine this text further.

Santa Monica Junior College. L. J. ADAMS.

Mathematical Analysis. By Maximilian Philip. Longmans, Green & Co., 1936. x+273 pages.

If orthodoxy is a virtue, *Mathematical Analysis* should be cast straightway into the pit. It is so thoroughly different from any book that the writer has seen before that his standards of comparison fail to function. Again and again in reading it he feels the warring within him of two impulses: one the protest of his rusted mental hinges against swinging doors backward, and the other his ingrained determination to be open-minded about the merits of the unaccustomed.

In 297 pages Mr. Philip achieves a look at an array of mathematical subjects imposing and formidable enough, ordinarily, to warrant dilution into six more or less harrowing installments. Differential calculus, plane analytic geometry, algebra, integral calculus, solid

analytics and trigonometry, all go into the grist—and the somewhat surprising order of the above items is not an accident of careless rhetoric. Trigonometry actually comes last, in so far as the orthodox subject-materials can be identified as units in the discussion.

The writer, however, does not wish to give the impression that the book is an unplanned hodge-podge of unrelated material. The unifying concept of the derivative, kept constantly in the foreground, knits into the common pattern each subject as it is introduced. The parts fit neatly into the whole, and the result is an interesting contribution to the literature of unified mathematics.

Except, however, for initially select groups, the book seems to this writer best adapted for a higher course to be taken by the judiciously decimated ranks of those who are ordinarily ready for "straight calculus." Though the author attempts, under his correlating pattern, to introduce all the usual subject matters on equal terms as if for the reader's first acquaintance, actually the book more nearly covers the conventional ground of calculus than of any other subject. The probably unwanted resemblance to a calculus text extends even to the proportions; integration is encountered on page 136, or just about halfway through. The usual freshmen subjects are introduced in just sufficient detail to show how the calculus methods can be applied to them also. This serves as a useful tie-in and review to those who have already been exposed for a reasonable time to the mysteries of algebra *et cetera*, whether tandem or abreast; but the writer is highly dubious about the perceptible effect, mathematically speaking, of the book's impact upon the stubborn and opaque resisting material in the mind of the average "unsorted" beginner.

In concept, nevertheless, the book is admirable. For an initial survey course restricted sharply to superior students, it should make an excellent text. One practical teaching objection seems inherent in its ambitious scope and small size—that of lack of completeness for the field covered. It gives more than a first look and less than a full treatment. This writer also finds a defect of execution in the choice of illustrative problem material. While the explanations are apparently carefully thought out, the examples themselves are not always happily chosen in the matter of direct and simple illustration of a principle. For instance, the derivative concept is first introduced in connection with the relatively complicated function, $y=2x+120/x$ (imagine the worst of the unsorted ones floundering with the deltas in that!)

Credit the author we must, certainly, with courage, independence, and stimulating originality. Debit him, probably, with a work which

would leave the transferring student in an unholy mess who would try to embark fresh from *Mathematical Analysis* to an old-fashioned institution where algebra is algebra. But this, perhaps, is more the fault of our educational hideboundness than of a fresh and intriguing work which blazes new trails.

Texas Technological College. R. S. UNDERWOOD.

The Study of The History of Mathematics. By George Sarton. Harvard University Press, Cambridge, 1936. 113 pp., cloth. $1.50.

This little manual will probably be found very useful to students of the history of mathematics, especially beginners. It contains an inaugural lecture of the author in a course on the history of mathematics at Harvard University on February 4, 1936. To this he has added a chapter on the study of the history of modern mathematics, properly distinguishing between the qualifications necessary for an historian of ancient mathematics in contrast to the modern field.

Our attention is called to the "bibliographical perversion", i. e., the practice of carrying this form of historical work to excess. Sarton furnishes us some useful "skeleton" bibliographies. Anyone interested in a particular item can easily pursue it further. This material falls under the headings of: General treatises, handbooks, treatises devoted to the history of special branches or mathematics, nineteenth and twentieth centuries, philosophy and methodology, bibliographical aids such as guides, encyclopedias and large catalogues, journals devoted to the history of mathematics, centers of research such as academies of science, mathematical societies, International Congresses (Proceedings), Institutes and special libraries.

Sarton feels that the biographical approach is the best for modern mathematics, and in the appendix furnishes a bibliography of biographies and related material on 118 mathematicians. We find listed the full biography where such exists, editions of the scholar's collected works, and editions of his correspondence. He might have stressed more strongly the use of primary sources, such as letters, manuscripts, etc., rather than printed sources. Neugebauer attaches less importance to priority questions than does Sarton, although the former is primarily interested in ancient mathematics. Neugebauer also feels less interested in the biographical side, it seems.

Above all, the reader will be glad that Dr. Sarton stresses the importance of historical *accuracy*. This volume will be a splendid addition to one's library and can certainly be read and referred to with profit. The indexes are well prepared, the printing and paper are excellent. Misprints are conspicuous by their absence.

University of Illinois. G. WALDO DUNNINGTON.

Fellowships in Science for Study in France for the Year 1837-1938

Administered by the

INSTITUTE OF INTERNATIONAL EDUCATION
2 West 45th Street
NEW YORK, NEW YORK

The Fondation Scientifique de Lyon du Sud-Est offers a fellowship of 10,000 francs for nine months at the University of Lyon to a graduate student of chemistry, preferably industrial chemistry. The student should have a knowledge of French sufficient to enable him to pursue his work with profit.

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The Office National des Universités et Ecoles Françaises is offering a fellowship of 18,000 francs for nine months study at any French university, free transportation to and from France (tourist class, French Line), and railroad fare from Le Havre to Paris and return to advanced students who have specialized in science, preferably men who have already obtained doctorates in mathematics, physical science, chemistry or biology.

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(a) Be American born.
(b) At the time of making the application be a graduate of a college, university, or professional school of recognized standing;
(c) Be of good moral character and intellectual ability, and of suitable personal qualities;
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(e) Possess ability to do independent study and research;
(f) Have a practical ability to use French books, both in general subjects and in his own special field, and be able to speak French and understand lectures delivered in French.

Preference in selection will be given to applicants between the ages of 25 and 35.

For application forms address the Student Bureau, Institute of International Education, 2 West 45th Street, New York City, sending ten cents to cover the cost. The closing date for filing applications with complete credentials is March 1, 1937.